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BUILDING AN ACCEPTANCE CHART AROUND THE PROCESS MEAN

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ABSTRACT

In this paper we will propose a new way of building an acceptance chart which could be utilized in Six Sigma environment. The proposed method is to build the acceptance chart around the desired value for the process mean rather than around the specification limits. An example is included to illustrate the proposed method.

Keywords: Acceptance charts, process control, process acceptance, Six Sigma program.

DISCUSSION

Process control charts, also known as Shewhart charts, deal with the process stability issue, i.e., they try to identify the types of variation (random (i.e., common) or special) that the process displays. A stable process is described as one that displays only a common cause of variation. Thus the question that is being tested on the control chart is "Is the process stable, i.e., in statistical control?" Quality practitioners like to keep their processes in stable condition so that they can predict their true capability and future performance and reduce variation and cost, etc., in the process. For more detailed information on control charts see, for example, Grant & Leavenworth [4], and Montgomery [8].

Acceptance charts, on the other hand, are designed to answer a different question. The question is not about the stability; it is about whether the process is producing parts that meet the customer specification limits. This question is very different from the one asked by the process control charts. Acceptance charts are suitable for processes, such as some chemical processes, that are expected to have unavoidable shifts in their average value but are still able to meet specifications set by the customer. In usual statistical process control terms, such a process is not considered in-control but may still be able to produce acceptable product. See, for example, Freund [3], Montgomery [8, pp.354-358], Holmes and Mergen [5, 6, 7] for more detailed explanations of the acceptance charts.

Acceptance charts are built around customer specification limits (one-sided or two-sided), using either 1) rejectable quality level and beta risk; 2) acceptable quality level and alpha risk; or 3) both rejectable and acceptable quality levels.

When the fluctuation of the process average is inevitable or allowed up to certain amount, acceptance charts are a good alternative, given that the process standard deviation is small enough compared to the width of the tolerance range, i.e., the difference between the upper and lower specification limits.

In this paper we will propose a new way of building an acceptance chart. The proposed method is to build the acceptance chart around the desired value for the process mean rather than around the specification limits; the desired value of the process mean could very well be the nominal value desired by the customer, if there is a nominal value. This proposed acceptance chart could be very useful, for example, in Six Sigma projects when the process mean is allowed to shift up to ± 1.5 standard deviations, i.e., it is assumed that the process average will not stay stable. In the Six Sigma environment, checking for the stability of the process mean may not be meaningful since the mean is allowed to shift. Thus the proposed acceptance chart would be a better alternative to check to see if the process average stays within the allowed range of fluctuation. The chart would help reduce the risk of stopping the process unnecessarily when the process mean stays within its allowed range of fluctuation. Use of conventional process control charts in this case, on the other hand, could give frequent and unnecessary "out-of-control" signals since they check a different question, i.e., "Has the process average shifted?" Thus, we are recommending the use of this proposed acceptance chart when the shift of the process mean is tolerated up to a certain level, such as the case in Six Sigma applications.

Statistically, Six Sigma means having no more than 3.4 defects per million opportunities in any process, product, or service. Pyzdek and Keller [9, page 3] describe Six Sigma as a program that incorporates elements from the work of many quality pioneers which aims for error-free business performance. The Six Sigma program involves the use of statistical tools within a structured methodology to gain the necessary knowledge to be more competitive by producing less expensive and better products/services faster and less expensively (Breyfogle [2] – also see Senturk, et al. [10] and Benitez et al. [1] for various Six Sigma applications). Those tools are applied through the DMAIC model: define the goals (D); measure the existing system (M); analyze the system to find ways to remove the gap between the current performance of the system and the desired performance (A); improve the system (I); and control (C) the new system (Pyzdek and Keller [9, pp.36-42]). The Six Sigma program, like other quality management programs, promotes being proactive rather than reactive. However, *"no more than 3.4 defects per million opportunities"* is valid under the assumption that the process average could shift up to ± 1.5 standard deviations, which yields about 4.5-sigma quality level. Six Sigma programs utilize many statistical, as well as some managerial tools. Based on our experience, however, acceptance charts are one of the least utilized tools in Six Sigma projects, despite the fact that conceptually these charts are very suitable for such projects given the assumption described above.

How to build the proposed acceptance chart:

The proposed acceptance charts will be based on a concept similar to the acceptable quality level, which will indicate how much fluctuation is desirable for the mean around the desired value, and the corresponding alpha risk. A constant, k_1, will determine this quantity, i.e., the amount of allowed fluctuation. In Six Sigma applications, since the process mean is allowed to shift up to ± 1.5 standard deviations, we will use $k_1=1.5$. Using $k_1=1.5$, the desired process average $(\bar{\bar{X}})$ and the process standard deviation (σ_X), the upper acceptable process mean (UAPM) value for the process mean will be calculated as follows (Figure 1):

$$UAPM = \bar{\bar{X}} + 1.5\sigma_X \qquad (1)$$

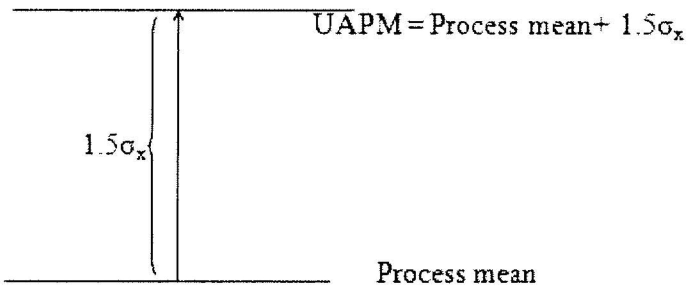

Figure 1. Graphical display of UAPM

This is the maximum value that we want to tolerate for the process mean. Then the upper acceptance limit (UAL) for the sample averages ($\bar{X}'s$) will be calculated as (Figure 2):

$$UAL = UAPM + k_2\sigma_{\bar{X}} \tag{2}$$

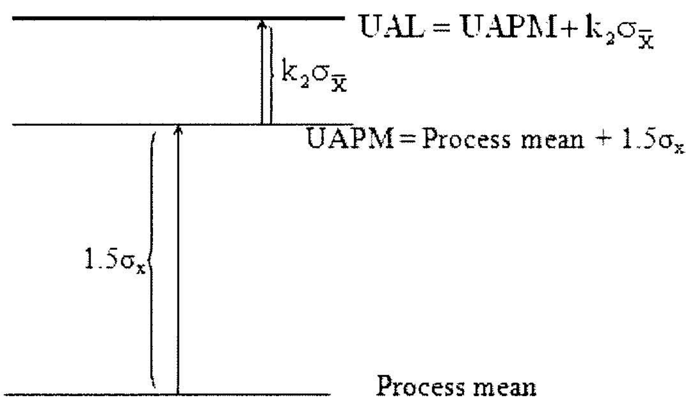

Figure 2. Graphical Display of UAL

where $\sigma_{\bar{X}}$ is the standard deviation of the sample averages ($\bar{X}'s$) and k_2 is a constant that would set the probability of accepting that the process average is not bigger than UAPM, which would also determine the resulting alpha risk (i.e., the type I error – rejecting the process when it is, in fact, running at the UAPM). In other words, we are addressing the question "Is the process behaving as the Six Sigma principle expects?" Remember that the standard deviation of the $\bar{X}'s$ is the standard deviation of the X's divided by the square root of the sample size, i.e., $\sigma_{\bar{X}} = \frac{\sigma_x}{\sqrt{n}}$ where n is the sample size.

Similar calculations could be done for the lowest acceptable process mean (LAPM) and the lower acceptance limit (LAL) as given below (Figure 3):

$$LAPM = \bar{\bar{X}} - 1.5\sigma_X \quad (3)$$

$$LAL = LAPM - k_2\sigma_{\bar{X}} \quad (4)$$

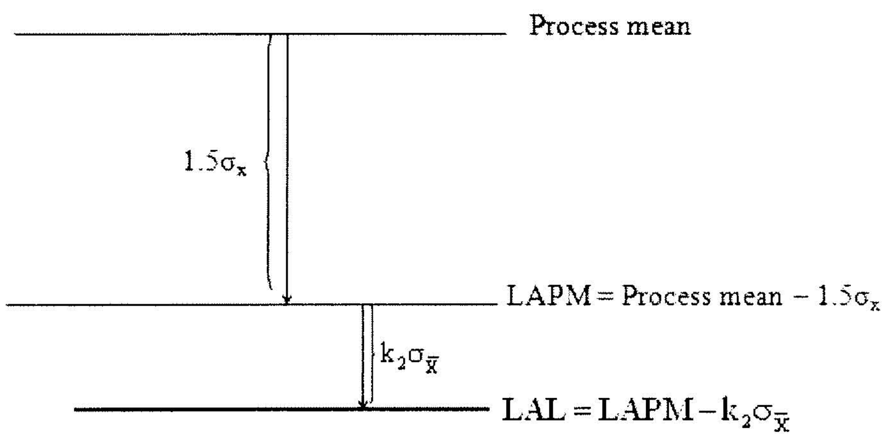

Figure 3. Graphical Display of LAPM and LAL

Thus, values of sample averages exceeding either UAL or LAL would be taken as a signal that the process average (mean) has shifted more than the allowed amount and that the proper corrective action should be taken. As long as the sample results fall between the acceptance limits, we will be accepting the fact that the process is behaving according to the Six Sigma model.

EXAMPLE

Let's assume that we have a process with a desired mean value of 50 and process output follows Normal distribution with standard deviation of 1. Let's also assume that process mean is allowed to fluctuate up to ±1.5 standard deviation around the desired value of 50. If we use k_2=3 and sample size (n) of 4, the acceptance limits for the proposed acceptance charts would be determined as follows:

$UAPM = 50 + 1.5(1) = 51.5$ $\qquad$ $UAL = 51.5 + 3\frac{1}{\sqrt{4}} = 53$

$LAPM = 50 - 1.5(1) = 48.5$ $\qquad$ $LAL = 48.5 - 3\frac{1}{\sqrt{4}} = 47$

Thus, as long as the sample averages fall between 47 and 53, we will accept (with 0.997 probability) that the process mean stays within ±1.5 standard deviation of the desired mean value of 50 (see also Figure 4 below). Thus no action would be required to adjust the mean.

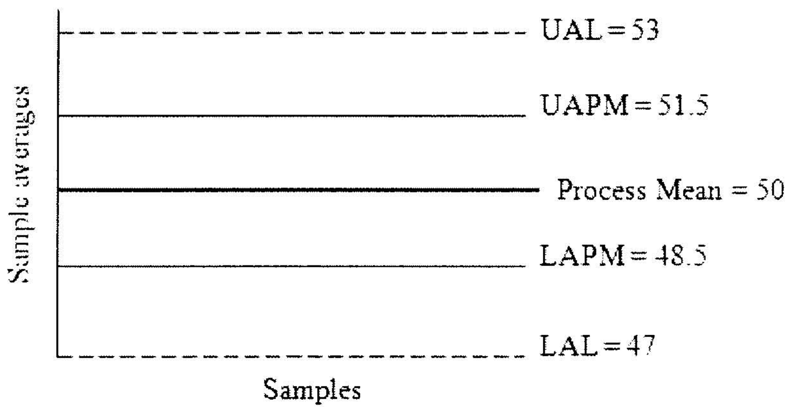

Figure 4. Acceptance chart around the mean.

CONCLUSION

In summary, in this paper we propose and describe a revised acceptance chart built around the process average to be used especially in Six Sigma projects when the process average is allowed to fluctuate to some degree. This proposed chart will help to keep the process average within the allowed range of fluctuation without increasing the alpha risk (i.e., the type I error). As a future study, a simulation analysis will be done to compare the false response rate of the control charts and the proposed acceptance chart when the shift in mean value is in the allowed range.

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